parameter and parameter space

exp(θ):

$X_1, \dots, X_n \overset{i.i.d}{\sim} \exp(\theta)$

p.d.f: $f(x) = \theta e^{-\theta x}$, $x \geq 0$.

$\theta \in (0, \infty)$, $\Omega = (0, \infty)$.

$N(\mu, \sigma^2)$

$X_1 \dots X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

p.d.f $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{-1}{2\sigma^2} (x-\mu)^2 \right\}$. , $x \in \mathbb{R}$

$\mu \in \mathbb{R}$, $\sigma^2 \in \mathbb{R}^+$

$(\mu, \sigma^2) \in \mathbb{R} \times \mathbb{R}^+$, $\Omega = \mathbb{R} \times \mathbb{R}^+$

Binomial (n, p)

$X_1 \dots X_n \overset{iid}{\sim} Bin(n, p)$

p.f. $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$, $x \in \{0, 1, \dots, n\}$

$n \in \mathbb{N}$, $p \in [0,1]$

$(n, p) \in \mathbb{N} \times [0,1]$, $\Omega = \mathbb{N} \times [0,1]$.

Example: diff. kinds of inferences:

X represents the daily temperatures.

a) $X \sim N(\mu, \sigma^2)$, $\theta = (\mu, \sigma^2)$ (theta)

p.d.f: $f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{1}{2\sigma^2}(x-\mu)^2 \right\}$, $x \in \mathbb{R}$

$\Omega = \mathbb{R} \times \mathbb{R}^+$.

b) P(the mean temperatures being less than 1.6)

$$Y = \frac{1}{n}\sum_{i=1}^{n} X_i$$

we know Y is going to have a normal distribution.

$$E(Y) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n}\mu = \mu$$

$$Var(Y) = Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} Var(\Sigma X_i) = \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i) = \frac{\sigma^2}{n}$$

$\therefore Y \sim N\left(\mu, \text{ variance} = \frac{\sigma^2}{n}\right)$

$P(Y < 1.6)$, $Y \sim N(\mu, \sigma^2/n)$