Classical and Bayesian inference

AMS 132

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Statistics and Estimators

Example

Assume that $X_1, \ldots, X_n$ form a random sample from the Bernoulli distribution with parameter θ, where each X_i describes if an iPhone is defective or not.

- Find, write, and plot the p.f. of $T = \sum_{i=1}^{n} X_i$ when $n = 30$.
- Find $P(T < 10 \mid \theta)$ and make a plot when $n = 30$.

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- The distribution of T is called the *sampling distribution* of the statistic T.
- The distribution of T can help us to solve questions as: how close we expect T to be to θ before observing any data; determine how much will we learn about θ by observing T, ...

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Definition (Sampling Distribution)

Suppose that the random variables $\boldsymbol{X} = (X_1, \ldots, X_n)$ form a random sample from a distribution involving a parameter θ whose value is unknown. Let T be a function of X and possibly θ. That is, $T = r(X_1, \ldots, X_n, \theta)$. The distribution of T (given θ) is called the *sampling distribution* of T. We will use the notation $E_\theta(T)$ to denote the mean of T calculated from its sampling distribution.

- The name "sampling distribution" comes from the fact that T depends on a random sample and so its distribution is derived from the distribution of the sample.
- When the random variable T does not depend on θ, it is a statistic, as previously defined.
- In principle, it is possible to derive the sampling distribution of each estimator of θ.

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Example

Consider that a large shipment of iPhone has arrive, and the proportion of defective iPhone, θ, is unknown. Assume that $X_1, \ldots, X_{10}$ for a random sample from the Bernoulli distribution with parameter θ.

- Find the M.L.E of θ, $\widehat{\theta}$, and an expression for $P(\mid \widehat{\theta} - \theta \mid < 0.1 \mid \theta)$.
- Find Bayes estimate of θ, $\delta^\star$, and an expression for $P(\mid \delta^\star - \theta \mid < 0.1 \mid \theta)$, when $\theta \sim Beta(10, 10)$.
- Plot both functions.

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Example (Sampling distribution of the M.L.E. of the mean of a normal distribution)

Consider a random sample $X_1, \ldots, X_n$ from the normal distribution with mean μ and known variance σ^2. Consider $n = 30$ and $\sigma^2 = 2$

- Find the sampling distribution of the M.L.E. of μ, $\widehat{\mu}$. Find an expression for $P(| \widehat{\mu} - \mu |< 3 \mid \mu)$.
- Find the sampling distribution of $\frac{\widehat{\mu}-\mu}{\sqrt{\sigma^2/n}}$. Find an expression for computing $P(| \widehat{\mu} - \mu | / \sqrt{\sigma^2/n} < 15)$.

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