Notes Class 2

$\xi(\theta)$

if Θ is a descrete parameter

$0 \le \xi(\theta) \le 1$, $\sum_{\theta \in \Omega} \xi(\theta) = 1$, $\xi(\theta)$ is a p.f. prior

if θ is a continious parameter

$0 \le \xi(\theta)$, $\int_{\Omega} \xi(\theta) d\theta = 1$, $\xi(\theta)$ is a p.d.f. prior

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Example: tossing a coin

a) statistical model:

X is a r.v that describes the result of tossing a coin.

$x \in \{0,1\}$

p.f $\quad f(x|\theta) = \theta^{x} (1-\theta)^{1-x}$

$\Omega = [0,1]$.

b) lets assume that $\theta \sim Beta(a,b)$

$$\xi(\theta) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \theta^{a-1} (1-\theta)^{b-1} \quad , \quad 0 \le \theta \le 1, \ a,b > 0.$$

if $a = b = 1$, $Beta(1,1) = U(0,1)$

c) $\xi(\theta = 0.5) = 0.8$ $\qquad \theta \in \{0.5, 1\}$

$\xi(\theta = 1) = 0.2$

Bayes Theorem (in sets)

A, B two sets, ~~Ω~~

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{\sum_A P(B|A)P(A)}$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{\sum_A P(A\cap B)}$$

Bayes Theorem

$$\xi(\theta|x_1 \ldots x_n) = \frac{\prod_{i=1}^{n} f(x_i|\theta)\ \xi(\theta)}{g_n(x_1 \ldots x_n)}$$

We have that $X_i|\theta \overset{iid}{\sim} F(\cdot|\theta)$ and the p.d.f of $X_i|\theta$ is $f(x_i|\theta)$. $\xi(\theta)$ is the prior ~~dens~~ p.d.f. of θ.

Note that

$$\xi(\theta|x_1 \ldots x_n) = \frac{f(x_1, \ldots x_n, \theta)}{g_n(x_1, \ldots, x_n)}$$

- We are going to get $f(x_1 \ldots x_n, \theta)$.

$$\begin{aligned} f(x_1, \ldots x_n, \theta) &= f(x_1 \ldots x_n|\theta)\xi(\theta) \\ &= f(x_1|\theta)f(x_2|\theta)\cdots f(x_n|\theta)\ \xi(\theta) \\ &\qquad \text{(by independence (cond. indep.))} \\ &= \prod_{i=1}^{n} f(x_i|\theta)\ \xi(\theta) \end{aligned}$$

- now we get $g_n(x_1 \ldots x_n)$

$$\begin{aligned} g_n(x_1 \ldots x_n) &= \int_\Omega f(x_1, \ldots x_n, \theta)\, d\theta = \\ &= \int \prod_{i=1}^{n} f(x_i|\theta)\cdot \xi(\theta)\, d\theta. \end{aligned}$$