

e) distr. of a new observation given $X_1=x_1, \dots, X_n=x_n$

$$f(x_{n+1} | x_1, \dots, x_n) = \int_0^{\infty} f(x_{n+1} | \theta) \xi(\theta | x) d\theta$$

$$= \int_0^{\infty} \frac{e^{-\theta} \theta^{x_{n+1}}}{x_{n+1}!} \frac{(75)^{194}}{P(194)} \theta^{194-1} e^{-75\theta} d\theta$$

$$= \frac{(75)^{194}}{P(194)} \frac{1}{x_{n+1}!} \int_0^{\infty} \theta^{194+x_{n+1}-1} e^{-76\theta} d\theta$$

Looks like Gamma (194 + x_{n+1}, 76)

~~$$= \frac{(75)^{194}}{P(194)} \frac{1}{x_{n+1}!} \frac{P(194+x_{n+1})}{(76)^{194+x_{n+1}}} \int_0^{\infty} \frac{76^{194+x_{n+1}}}{P(194+x_{n+1})} \theta^{194+x_{n+1}-1} e^{-76\theta} d\theta$$~~

$$= \frac{(75)^{194}}{P(194)} \frac{1}{x_{n+1}!} \frac{P(194+x_{n+1})}{(76)^{194+x_{n+1}}} \underbrace{\int_0^{\infty} \frac{76^{194+x_{n+1}}}{P(194+x_{n+1})} \theta^{194+x_{n+1}-1} e^{-76\theta} d\theta}_1$$

$$= \frac{75^{194}}{76^{194+x_{n+1}}} \frac{P(194+x_{n+1})}{P(194)} \frac{1}{x_{n+1}!}$$