University of California, Santa Cruz
Department of Applied Mathematics and Statistics
Baskin School of Engineering
Classical and Bayesian Inference - AMS 132

Exam 2: Class, Version A

Name:

Instructions: Write your name in every page. This is a closed-book, closed-notes exam, with the exception of one (letter size) piece of paper with formulas on both sides. Note that last page includes some distributions and other results that are useful for these problems. You can use without proof any result developed in class, homework, or included in the textbook sections that correspond to the material for this exam. Any other result you use for a problem solution must be derived. Please show all your work, detail the steps for solving each problem, and indicate your answers in the spaces provided. If you need more space, ask for an extra page. Please note that unsupported answers will receive little (or no) credit, even if they are correct. **Good luck!**.

1. Suppose that $X_1, \dots, X_n$ form a random sample from the normal distribution with mean μ and variance σ^2, and let $\widehat{\sigma_0}^2 = \frac{1}{n}\sum_{i=1}^n (X_i - \mu)^2$. **[6 pts]**

   (a) Find the distribution of $\frac{n\widehat{\sigma_0}^2}{\sigma^2}$. Explain in detail how you get this distribution, this is, explain in detail how starting from considering $X_i \sim N(\mu, \sigma^2)$ independently, you get the distribution of $\frac{n\widehat{\sigma_0}^2}{\sigma^2}$. **[3 pts]**

   First, notice that
   $$\frac{n\widehat{\sigma_0}^2}{\sigma^2} = \frac{n \times \frac{1}{n}\sum_{i=1}^n (X_i-\mu)^2}{\sigma^2} = \sum_{i=1}^n \left(\frac{X_i-\mu}{\sigma}\right)^2.$$
   Now, since $X_i \sim N(\mu, \sigma^2)$, and are independent, it follows that $\left(\frac{X_i-\mu}{\sigma}\right) \sim N(0,1)$, and are independent. This implies that $\left(\frac{X_i-\mu}{\sigma}\right)^2 \sim \chi^2_{(1)}$, and are independent, which in turn implies that $\sum_{i=1}^n \left(\frac{X_i-\mu}{\sigma}\right)^2 \sim \chi^2_{(n)}$. Therefore, $\frac{n\widehat{\sigma_0}^2}{\sigma^2} \sim \chi^2_{(n)}$.

   (b) Assume that a sample of size 12 is considered. Find the value c such that $P\left(\frac{\widehat{\sigma_0}^2}{\sigma^2} \le c\right) = 0.95$. **[3 pts]**

   Since a sample of size 12 is considered, $\frac{n\widehat{\sigma_0}^2}{\sigma^2} \sim \chi^2_{(12)}$. Now, $P\left(\frac{\widehat{\sigma_0}^2}{\sigma^2} \le c\right) = P(Y \le cn) = P(Y \le 12c)$, where $Y \sim \chi^2_{(12)}$. So, we need to find the 0.95 quantile of a χ^2 distribution with 12 degrees of freedom. From the values ate the end of exam, this quantile is 21.0260. So $12c = 21.0260$ implies that $c = 21.0260/12 = 1.7521$.

2. Suppose that $X_1, \ldots, X_n$ are random variables describing the calorie content of beef hot dogs from n brands. Assume that they are independent normally distributed random variables with mean μ and variance σ^2, with know variance equal to 510. It is of interest to find a 90% confidence interval for the mean number of calories of these beef hot dog brands. **[12 pts]**

   (a) Find the smallest sample size that should be considered to have a 90% confidence interval for the mean number of calories of length equal or smaller that 16.61. **[4 pts]**

   Since the variance is known, the 90% confidence interval for the mean number of calories is given by

$$\left(\overline{X}_n - \Phi^{-1}\left(\frac{1.9}{2}\right)\frac{\sigma}{\sqrt{n}}, \overline{X}_n + \Phi^{-1}\left(\frac{1.9}{2}\right)\frac{\sigma}{\sqrt{n}}\right),$$

   where $\sigma = \sqrt{510} = 22.58$ Therefore, the length L of the interval is $L = 2\Phi^{-1}\left(\frac{1.9}{2}\right)\frac{\sigma}{\sqrt{n}} = \frac{2*1.64*22.58}{\sqrt{n}}$. So for the interval to be of length equal or smaller that 16.61, we need $\frac{2*1.64*22.58}{\sqrt{n}} = \frac{74.06}{\sqrt{n}} \leq 16.61$. Therefore, $n \geq \left(\frac{74.06}{16.61}\right)^2 = 19.880$. So, $n = 20$ is given a 90% confidence interval.

   (b) In the June 1986 issue of *Consumer Reports*, the numbers of calories in 20 different beef hot dog brands were reported, showing that $\overline{x}_n = 156.85$. Compute 90% confidence interval for the mean number of calories. **[4 pts]**

   Here we compute the value of the interval

$$(A, B) = \left(\overline{X}_n - \Phi^{-1}\left(\frac{1.9}{2}\right)\frac{\sigma}{\sqrt{n}}, \overline{X}_n + \Phi^{-1}\left(\frac{1.9}{2}\right)\frac{\sigma}{\sqrt{n}}\right).$$

   Here $a = \overline{x}_n - \Phi^{-1}\left(\frac{1.9}{2}\right)\frac{\sigma}{\sqrt{n}} = 156.85 - 1.64 * \frac{sqrt510}{\sqrt{20}} = 148.56$, and $b = \overline{x}_n + \Phi^{-1}\left(\frac{1.9}{2}\right)\frac{\sigma}{\sqrt{n}} = 156.85 + 1.64 * \frac{sqrt510}{\sqrt{20}} = 165.13$. So, the 90% confidence interval for the mean number of calories is $(148.56, 165.13)$.

   (c) A beef hot dog is considered healthy if it has around than 150 calories. Would you say these brands produce healthy beef hot dogs? **[4 pts]**

   Since 150 calories is within the interval, we are 90% confident that these brands produce healthy beef hot dogs.

3. Let $X_1, \ldots, X_n$ be a random sample from the normal distribution with mean μ_1 and known variance $\sigma^2 = 0.49$, where X_i describe the body temperature of a female. Let $Y_1, \ldots, Y_m$ be a random sample from the normal distribution with mean μ_2 and known variance $\sigma^2 = 0.49$, where Y_i describes the body temperature of a male. The interest is to know if there is a difference between the mean body temperatures of males and females. For this we will compute a symmetric 90% confidence interval for the difference between the mean body temperatures of males and females. **[12 pts]**

   (a) Show that
$$Z = \frac{\overline{X}_n - \overline{Y}_n - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}}$$
   is a pivot that has the standard normal distribution. Explain in detail how you get this distribution. **[4 pts]**

   Since $X_i \sim N(\mu_1, \sigma^2)$ and $Y_i \sim N(\mu_2, \sigma^2)$, and they are independent, it follows that $\overline{X}_n \sim N(\mu_1, \sigma^2/n)$, and $\overline{Y}_m \sim N(\mu_2, \sigma^2/m)$ and they are independent. So, $\overline{X}_n - \overline{Y}_m \sim N(\mu_1 - \mu_2, \sigma^2/n + \sigma^2/m)$. Then, standardizing this last random variable, it follows that
$$Z = \frac{\overline{X}_n - \overline{Y}_n - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}} \sim N(0, 1)$$

   (b) Find random variables A and B such that $P(A < \mu_1 - \mu_2 < B) = \gamma$, where γ is the coefficient of the symmetric confidence interval. **[4 pts]**
   Note: for finding A and B you can start from $P(G^{-1}(\gamma_1) < Z < G^{-1}(\gamma_2))$, where $G^{-1}(\gamma_1)$ and $G^{-1}(\gamma_2)$ are the γ_1 and γ_2 quantiles of the distribution of Z.

   Since $Z \sim N(0, 1)$ and we are interested in a symmetric 90% confidence interval, we can choose $\gamma_1 = 0.05$ and $\gamma_2 = 0.95$, so that $G^{-1}(\gamma_1) = \Phi^{-1}(0.05) = -\Phi^{-1}\left(\frac{1.9}{2}\right)$, and $G^{-1}(\gamma_2) = \Phi^{-1}(0.95) = \Phi^{-1}\left(\frac{1.9}{2}\right)$. Now,

$$P(G^{-1}(\gamma_1) < Z < G^{-1}(\gamma_2)) = P\left(-\Phi^{-1}\left(\frac{1.9}{2}\right) < \frac{\overline{X}_n - \overline{Y}_n - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}} < \Phi^{-1}\left(\frac{1.9}{2}\right)\right),$$

$$= P\left(-\Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n} + \frac{1}{m}} < \overline{X}_n - \overline{Y}_n - (\mu_1 - \mu_2) < \Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}\right),$$

$$= P\left(-(\overline{X}_n - \overline{Y}_n) - \Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n} + \frac{1}{m}} < -(\mu_1 - \mu_2) < -\overline{X}_n - \overline{Y}_n + \Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}\right),$$

$$= P\left(\overline{X}_n - \overline{Y}_n - \Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n} + \frac{1}{m}} < \mu_1 - \mu_2 < \overline{X}_n - \overline{Y}_n + \Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}\right),$$

So, the random variables A and B are given by

$$A = \overline{X}_n - \overline{Y}_n - \Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n}+\frac{1}{m}}$$

and

$$B = \overline{X}_n - \overline{Y}_n + \Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n}+\frac{1}{m}}.$$

(c) From a sample of size 65 of females it is observed that $\overline{x}_{65} = 98.1$. From a sample of size 70 of males, it is observed that $\overline{y}_{70} = 98.8$. Compute the 90% confidence interval for the difference between the mean body temperatures of males and females. Can you say that there a difference between the mean body temperatures of males and females? **[4 pts]**

Here we compute the observed values of A and B. Here,

$$a = \overline{x}_n - \overline{y}_n - \Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n}+\frac{1}{m}} = 98.1 - 98.9 - 1.64 * sqrt0.49 * \sqrt{1/65+1/70} = -0.8983,$$

$$b = \overline{x}_n - \overline{y}_n + \Phi^{-1}\left(\frac{1.9}{2}\right)\sigma\sqrt{\frac{1}{n}+\frac{1}{m}} = 98.1 - 98.9 + 1.64 * sqrt0.49 * \sqrt{1/65+1/70} = -0.5016.$$

Since 0 is not within the interval, with a 90% confidence we can say that there is a difference between the mean body temperatures of males and females, males presenting higher temperatures that females.

Exam 2: Take home, Version A

Name:

Instructions: You can use `R` or a table to get any quantile or probability you need, but be clear of what distribution are you considering, what are you computing (a quantile or a probability), how many degrees of freedom, etc. **Good luck!**.

Let $X_1, \ldots, X_n$ be random variables that describe the income of n subjects in the city of Santa Cruz, and let $Y_1, \ldots, Y_m$ be random variables that describe the income of m subjects in the city of San José. Assume that $X_1, \ldots, X_n$ from a random sample from the normal distribution with mean μ_1 and variance σ^2, where both μ_1 and σ^2 are unknown, and independently, assume that $Y_1, \ldots, Y_m$ from a random sample from the normal distribution with mean μ_2 and variance σ^2, where both μ_2 and σ^2 are unknown.

1. We are interested in comparing the mean income received by people in Santa Cruz and San José. For this, we are going to compute a symmetric 90% confidence interval for the difference between the mean income received in Santa Cruz and the mean income received in San José. **[15 pts]**

   (a) Show that
$$W = \frac{\overline{X}_n - \overline{Y}_m - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{1}{n} + \frac{1}{m}\right) \frac{\sum_{i=1}^{n}(X_i - \overline{X}_n)^2 + \sum_{i=1}^{m}(Y_i - \overline{Y}_m)^2}{n+m-2}}}$$
is a pivot that has the t distribution with $n + m - 2$ degrees of freedom. **[5 pts]**
Note: for showing this, use the fact that if $Z_1 \sim \chi^2_{(r)}$ and $Z_2 \sim \chi^2_{(r)}$, and they are independent, then $Z_1 + Z_2 \sim \chi^2_{(r+s)}$.
Since $X_i \sim N(\mu_1, \sigma^2)$ and $Y_i \sim N(\mu_2, \sigma^2)$, and they are independent, it follows that $\overline{X}_n \sim N(\mu_1, \sigma^2/n)$, and $\overline{Y}_m \sim N(\mu_2, \sigma^2/m)$ and they are independent. So, $\overline{X}_n - \overline{Y}_m \sim N(\mu_1 - \mu_2, \sigma^2/n + \sigma^2/m)$. Then, standardizing this last random variable, it follows that
$$\frac{\overline{X}_n - \overline{Y}_n - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}} \sim N(0, 1).$$
Now, since $X_i \sim N(\mu_1, \sigma^2)$ and $Y_i \sim N(\mu_2, \sigma^2)$, and they are independent, by the theorem of joint distribution for the sample mean and sample variance, it follows that $\sum_{i=1}^{n}\left(\frac{X_i - \overline{X}_n}{\sigma}\right)^2 \sim \chi_{(n-1)}$ and $\sum_{i=1}^{n}\left(\frac{Y_i - \overline{Y}_m}{\sigma}\right)^2 \sim \chi_{(m-1)}$, and they are independent. Therefore, by the note, it follows that
$$\frac{1}{\sigma^2}\left[\sum_{i=1}^{n}(X_i - \overline{X}_n)^2 + \sum_{i=1}^{m}(Y_i - \overline{Y}_m)^2\right] \sim \chi^2_{(n+m-2)}.$$
Finally,
$$\frac{\frac{\overline{X}_n - \overline{Y}_n - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n} + \frac{1}{m}}}}{\sqrt{\frac{1}{\sigma^2}\left[\sum_{i=1}^{n}(X_i - \overline{X}_n)^2 + \sum_{i=1}^{m}(Y_i - \overline{Y}_m)^2\right]/(n+m-2)}}$$

$$= \frac{\overline{X}_n - \overline{Y}_m - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{1}{n} + \frac{1}{m}\right) \frac{\sum_{i=1}^n (X_i - \overline{X}_n)^2 + \sum_{i=1}^m (Y_i - \overline{Y}_m)^2}{n+m-2}}} \sim t_{n+m-2}.$$

(b) Find a symmetric coefficient γ confidence interval for $\mu_1 - \mu_2$ using the result from question 1. a). For this, find random variables A and B such that $P(A < \mu_1 - \mu_2 < B) = \gamma$. **[5 pts]**

Since we are interested in a symmetric 90% confidence interval, we choose $\gamma_1 = 0.05$ and $\gamma_2 = 0.95$. Now, since W has a t distribution with $n + m - 2$ degrees of freedom, the quantiles that we are interested in are $G^{-1}(\gamma_1) = T^{-1}_{m+n-2}(0.05) = -T^{-1}_{m+n-2}(1.9/2)$, and $G^{-1}(\gamma_2) = T^{-1}_{m+n-2}(0.95) = T^{-1}_{m+n-2}(1.9/2)$. Notice that

$$P(G^{-1}(\gamma_1) < W < G^{-1}(\gamma_2)) = P(-T^{-1}_{m+n-2}(1.9/2) < W < T^{-1}_{m+n-2}(1.9/2)) = 0.9.$$

From here, it follows that

$$P(-T^{-1}_{m+n-2}(1.9/2) < \frac{\overline{X}_n - \overline{Y}_m - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{1}{n} + \frac{1}{m}\right) \frac{\sum_{i=1}^n (X_i - \overline{X}_n)^2 + \sum_{i=1}^m (Y_i - \overline{Y}_m)^2}{n+m-2}}} < T^{-1}_{m+n-2}(1.9/2)) = 0.9$$

, and that a symmetric 90% confidence interval for $\mu_1 - \mu_2$ is given by (A, B), where

$$A = \overline{X}_n - \overline{Y}_m - T^{-1}_{m+n-2}(1.9/2)\sqrt{\left(\frac{1}{n} + \frac{1}{m}\right) \frac{\sum_{i=1}^n (X_i - \overline{X}_n)^2 + \sum_{i=1}^m (Y_i - \overline{Y}_m)^2}{n+m-2}}$$

and

$$B = \overline{X}_n - \overline{Y}_m + T^{-1}_{m+n-2}(1.9/2)\sqrt{\left(\frac{1}{n} + \frac{1}{m}\right) \frac{\sum_{i=1}^n (X_i - \overline{X}_n)^2 + \sum_{i=1}^m (Y_i - \overline{Y}_m)^2}{n+m-2}}$$

(c) From a sample of 26 subjects living in Santa Cruz, it was observed that the sample mean is 5.80 (thousands of dollars), and $\sum_{i=1}^{26} (x_i - \overline{x}_n)^2 = 217.13$. From a sample of size 21 subjects living in San José, it was observed that the sample mean is 6.17 (thousands of dollars), and $\sum_{i=1}^{21} (y_i - \overline{y}_n)^2 = 125.11$. Compute a symmetric 90% confidence interval for $\mu_1 - \mu_2$. Can you say that there is a difference in income for people living in Santa Cruz and San José? **[5 pts]**

Here we compute the observed symmetric 90% cofidence interval for the difference of the means

$$a = 5.80 - 6.17 - 1.6794\sqrt{\left(\frac{1}{26} + \frac{1}{21}\right) \frac{217.13 + 125.11}{45}} = -1.7288$$

$$b = 5.80 - 6.17 + 1.6794\sqrt{\left(\frac{1}{26} + \frac{1}{21}\right) \frac{217.13 + 125.11}{45}} = 0.9888.$$

With a 90% confidence we can say that there is no difference in the income that people in Santa Cruz and San José receive.

2. In question 1. we assumed that both populations, income of people in Santa Cruz and income of people in San José, have the same unknown variance, but this assumption might be wrong. For this,

we are going to compute a symmetric 90% confidence interval for the ratio of the variances. Assume that $X_1, \ldots, X_n$ from a random sample from the normal distribution with mean μ_1 and variance σ_1^2, where both μ_1 and σ_1^2 are unknown, and independently, assume that $Y_1, \ldots, Y_m$ from a random sample from the normal distribution with mean μ_2 and variance σ_2^2, where both μ_2 and σ_2^2 are unknown. **[15 pts]**

(a) Show that

$$W = \frac{\sigma_2^2}{\sigma_1^2} \frac{\sum_{i=1}^{n}(X_i - \overline{X}_n)^2/(n-1)}{\sum_{i=1}^{m}(Y_i - \overline{Y}_m)^2/(m-1)}$$

is a pivot that has a Fisher distribution with $n-1$ and $m-1$ degrees of freedom. **[5 pts]**
Note: for showing this, use the fact that if $Z_1 \sim \chi^2_{(r)}$ and $Z_2 \sim \chi^2_{(s)}$, and they are independent, then $\frac{Z_1/r}{Z_2/s}$ has a Fisher distribution with r and s degrees of freedom.

Since $X_i \sim N(\mu_1, \sigma_1^2)$ and $Y_i \sim N(\mu_2, \sigma_2^2)$, and they are independent, by the theorem of joint distribution of the sample men and sample variance, it follows that $\sum_{i=1}^{n}\left(\frac{X_i - \overline{X}_n}{\sigma_1}\right)^2 \sim \chi^2_{(n-1)}$ and $\sum_{i=1}^{m}\left(\frac{Y_i - \overline{Y}_m}{\sigma_2}\right)^2 \sim \chi^2_{(m-1)}$. Therefore, by the note, it follows that

$$\frac{\sum_{i=1}^{n}\left(\frac{X_i - \overline{X}_n}{\sigma_1}\right)^2/(n-1)}{\sum_{i=1}^{m}\left(\frac{Y_i - \overline{Y}_m}{\sigma_2}\right)^2/(m-1)} = \frac{\sigma_2^2}{\sigma_1^2} \frac{\sum_{i=1}^{n}(X_i - \overline{X}_n)^2/(n-1)}{\sum_{i=1}^{m}(Y_i - \overline{Y}_m)^2/(m-1)} \sim F_{n-1,m-1},$$

where $F_{n-1,m-1}$ denotes a Fisher distribution with $n-1$ and $m-1$ degrees of freedom.

(b) Find a symmetric 90% confidence interval for $\frac{\sigma_2^2}{\sigma_1^2}$ using the result from question 2. a). For this, find random variables A and B such that $P(A < \frac{\sigma_2^2}{\sigma_1^2} < B) = 0.9$. **[5 pts]**
Note: for finding A and B you can start from $P(G^{-1}(0.05) < W < G^{-1}(0.95))$, where $G^{-1}(0.05) = F^{-1}_{n-1,m-1}(0.05)$ and $G^{-1}(0.95) == F^{-1}_{n-1,m-1}(0.95)$ are the 0.05 and 0.95 quantiles of the distribution of W.

Since we are interested in a symmetric 90% confidence interval for the ratio of the variance, we consider $\gamma_1 = 0.05$ and $\gamma_2 = 0.95$, so $G^{-1}(0.05)$ and $G^{-1}(0.95)$ are the 0.05 and 0.95 quatniles of a Fisher distribution with $n-1$ and $m-1$ degrees of freedom, respectively. Note that $P(G^{-1}(0.05) < W < G^{-1}(0.95)) = \gamma$ and is equivalent to

$$P\left(G^{-1}(0.05) < \frac{\sigma_2^2}{\sigma_1^2} \frac{\sum_{i=1}^{n}(X_i - \overline{X}_n)^2/(n-1)}{\sum_{i=1}^{m}(Y_i - \overline{Y}_m)^2/(m-1)} < G^{-1}(0.95)\right) = \gamma.$$

From the above, we can find a symmetric 90% confidence interval (A, B), where

$$A = \frac{\sum_{i=1}^{m}(Y_i - \overline{Y}_m)^2/(m-1)}{\sum_{i=1}^{n}(X_i - \overline{X}_n)^2/(n-1)} F^{-1}_{n-1,m-1}(0.05)$$

and

$$B = \frac{\sum_{i=1}^{m}(Y_i - \overline{Y}_m)^2/(m-1)}{\sum_{i=1}^{n}(X_i - \overline{X}_n)^2/(n-1)} F^{-1}_{n-1,m-1}(0.95).$$

(c) Using 2. b), compute a symmetric 90% confidence interval for $\frac{\sigma_2^2}{\sigma_1^2}$. Can you say that the equal variances assumption in question 1. is reasonable? **[5 pts]**

Note: for computing the 0.05 and 0.95 quantiles of a Fisher distribution with r and s degrees of freedom, you can use the `R` command `qf(0.05, df1=r, df2=s)` and `qf(0.95, df1=r, df2=s)`.

Here we compute the observed 90% symmetric confidence interval (a, b), where

$$a = \frac{125.11/20}{217.13/25} 0.4821 = 0.34728$$

and

$$b = \frac{125.11/20}{217.13/25} 2.0074 = 1.4458.$$

So, with a 90% confidence, we have that the ratio of the variances, $\frac{\sigma_2^2}{\sigma_1^2}$ is equal to 1, and the equal variances assumption for the income of people in Santa Cruz and San José is reasonable.

Name:

Some quantiles:

```
> qchisq(0.90, df=11)    > qchisq(0.95, df=11)    > qchisq(0.99, df=11)
[1] 17.2750              [1] 19.6751              [1] 24.7249
> qchisq(0.90, df=12)    > qchisq(0.95, df=12)    > qchisq(0.99, df=12)
[1] 18.5493              [1] 21.0260              [1] 26.2169
> qchisq(0.90, df=13)    > qchisq(0.95, df=13)    > qchisq(0.99, df=13)
[1] 19.8119              [1] 22.3620              [1] 27.6882
```

```
> qnorm(0.90)       > qnorm(0.95)      > qnorm(0.99)
[1] 1.281552         [1] 1.6448        [1] 2.3263
> qnorm(0.10)       > qnorm(0.05)      > qnorm(0.01)
[1] -1.281552        [1] -1.6448       [1] -2.3263
```