These questions were covered in details during the discussion sections.

1. Suppose that $X_1, ..., X_n$ form a random sample from the normal distribution with unknown mean μ and **unknown** standard deviation σ, and let $\hat{\mu}$ and $\hat{\sigma}$ denote the M.L.E.'s of μ and σ. For sample size $n = 17$, find a value of k such that

   $$P[\hat{\mu} > \mu + k\hat{\sigma}] = 0.95.$$

2. Suppose that $X_1, ..., X_n$ form a random sample from the normal distribution with mean μ and standard deviation σ, and Y is an independent random variable having the normal distribution with mean 0 and variance $4\sigma^2$. Determine a function of $X_1, ..., X_n$ and Y that does not involve μ and σ^2, but has the t distribution with $n - 1$ degrees of freedom.

3. Suppose that $X_1, ..., X_n$ form a random sample from the normal distribution with unknown mean μ and **known** standard deviation σ. Let Φ stand for the c.d.f. of the standard normal distribution, and let Φ^{-1} be its inverse. Show that the following interval is coefficient γ confidence interval for μ if $\overline{X}_n$ is the observed average of the data values:

   $$\left(\overline{X}_n - \Phi^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma}{n^{1/2}}, \overline{X}_n + \Phi^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma}{n^{1/2}}\right)$$

4. Suppose that $X_1, ..., X_n$ form a random sample from the normal distribution with unknown mean μ and **unknown** standard deviation σ. Construct a upper confidence limit $U(X_1, ..., X_n)$ for σ^2 such that

   $$P[\sigma^2 < U(X_1, ..., X_n)] = 0.99.$$