

These questions were covered in details during the discussion sections.

1. Suppose that X_1, \dots, X_n form a random sample from a normal distribution for which the value of the parameters μ and σ^2 are unknown. Find the M.L.E. of μ and σ^2 .
2. Let X be a random variable following an exponential distribution with unknown parameter β . Find the moment generating function of X and use it to compute the second moment of X .
3. Let X and Y be independent random variables with the following distributions:

$$X \sim N(0, 1)$$

$$Y \sim N(3, 5)$$

Find the distribution of the following random variables:

- (a) $X + 2$
 - (b) $3Y$
 - (c) $X + Y$
 - (d) $X - Y$
 - (e) $aX + b$
 - (f) $aX + bY$
4. Let Y be a random variable with the following distribution:

$$Y \sim N(3, 5).$$

Find the following probabilities:

- (a) $P(Y < 3)$
 - (b) $P(Y < 6)$
 - (c) $P(3Y < 2)$
5. Suppose that the p.d.f. of a random variable X is as follows:

$$f(x) = \frac{x}{2}, \quad 0 < x < 2$$

and 0 otherwise. Determine the p.d.f of $Y = 4 - X^3$

6. Suppose that X_1 and X_2 have a continuous joint distribution for which the joint p.d.f. is:

$$f(x_1, x_2) = 2(x_1 + x_2), \quad 0 < x_1 < x_2 < 1$$

and 0 otherwise. Determine the p.d.f. of $Z = X_1 + X_2$.