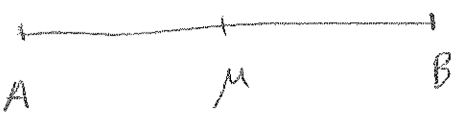
$$A = \bar{X}_n - T_{n-1}^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma'}{\sqrt{n}}$$

$$B = \bar{X}_n + T_{n-1}^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma'}{\sqrt{n}}$$

(A, B) is is $100 \cdot \gamma$ percent confidence interval for μ.



Example: lactic acid concentration in cheese.

a) $X_i \overset{iid}{\sim} N(\mu, \sigma^2)$. Find a 90 percent conf. interval:

The 90 percent confidence interval is given by (A, B), where

$$A = \bar{X}_n - T_{n-1}^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma'}{\sqrt{n}}$$

$$B = \bar{X}_n + T_{n-1}^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma'}{\sqrt{n}}.$$

$n = 10.$

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\sigma' = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X}_n)^2}{n-1}}$$

$\gamma = 0.9.$

$$T_{n-1}^{-1}\left(\frac{1+\gamma}{2}\right) = T_9^{-1}\left(\frac{1+0.9}{2}\right) = T_9^{-1}\left(\frac{1.9}{2}\right) = T_9^{-1}(0.95) = 1.83$$

$$A = \bar{X}_n - 1.83 \cdot \frac{\sigma'}{\sqrt{10}}$$

$$B = \bar{X}_n + 1.83 \frac{\sigma'}{\sqrt{10}}.$$

b) $a = \bar{x}_n - 1.83 \dfrac{\sigma'}{\sqrt{10}} = 1.379 - 1.83 \cdot 0.327 = 1.189$

$b = \bar{x}_n + 1.83 \dfrac{\sigma'}{\sqrt{10}} = 1.379 + 1.83 \cdot 0.327 = 1.568.$

One-sided interval: <u>lower confidence interval.</u>

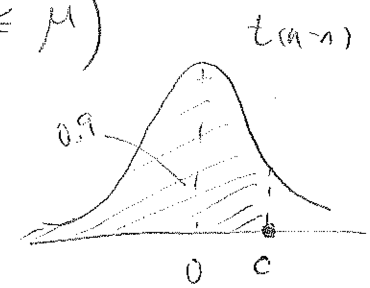
$X_i \sim N(\mu, \sigma^2)$, μ, σ^2 unknown

$U = \dfrac{\sqrt{n}\,(\bar{X}_n - \mu)}{\sigma'}$ has a t distr. with $n-1$ degrees of freedom.

$P(U \leq c) = \gamma$

$$P\left(\frac{\sqrt{n}\,(\bar{X}_n - \mu)}{\sigma'} \leq c\right) = P\left(\bar{X}_n - \mu \leq \frac{c\sigma'}{\sqrt{n}}\right)$$

$$= P\Big(\underbrace{\bar{X}_n - c\,\frac{\sigma'}{\sqrt{n}}}_{A} \leq \mu\Big)$$



$c = T_{n-1}^{-1}(\gamma)$ $\quad\quad \gamma = 0.9$

(A, ∞)

<u>Upper confidence interval:</u> ......

if $X_1 \dots X_n$ form a random sample from $N(\mu, \sigma^2)$

$$\underbrace{\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma'}} \sim t_{n-1}$$

is a pivotal : $V(\underset{\sim}{X}, \mu) = \frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma'}$

$$\underbrace{\sum_{i=1}^{n} \left(\frac{X_i - \bar{X}_n}{\sigma}\right)^2} \sim \chi^2_{(n-1)}$$

is a pivotal : $V(\underset{\sim}{X}, \sigma) = \sum_{i=1}^{n} \left(\frac{X_i - \bar{X}_n}{\sigma}\right)^2$

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if $V(\underset{\sim}{X}, \mu) = \frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma'}$

$$\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma'} = c$$

$$\bar{X}_n - \mu = \frac{\sigma' c}{\sqrt{n}}$$

$$\mu = \bar{X}_n - \frac{\sigma' c}{\sqrt{n}}$$

if $V(\underset{\sim}{X}, \sigma^2) = \sum_{i=1}^{n} \left(\frac{X_i - \bar{X}_n}{\sigma}\right)^2$

$$\sum_{i=1}^{n} \left(\frac{X_i - \bar{X}_n}{\sigma}\right)^2 = c$$

$$\frac{1}{\sigma^2} \sum (X_i - \bar{X}_n)^2 = c$$

$$\sigma^2 = \left[\frac{c}{\sum (X_i - \bar{X}_n)^2}\right]^{-1}$$

if $V(X, \sigma^2, \mu) = \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2$

$$\frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \mu)^2 = c$$

$$\sigma^2 = \left[\frac{c}{\sum (X_i - \mu)^2}\right]^{-1} \longleftarrow \text{depends on } \mu.$$