Right / Wrong assumptions:

b) $X_i \sim N(\mu, \sigma^2)$, σ^2 unknown.

Under these assumptions we know that a 90% conf interval for μ is given by

$$(A, B) = \left(\overline{X_n} - T_{n-1}^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma'}{\sqrt{n}} , \overline{X_n} + T_{n-1}^{-1}\left(\frac{1+\gamma}{2}\right)\frac{\sigma'}{\sqrt{n}} \right), \gamma = 0.9$$

$n = 25$

$$\sigma' = \sqrt{\frac{\sum (x_i - \overline{x_n})^2}{n-1}} = 6.10.$$

$$\overline{x_n} = \frac{1}{n}\sum_{i=1}^{n} x_i = 5.77$$

$$T_{n-1}^{-1}\left(\frac{1+\gamma}{2}\right) = T_{n-1}^{-1}\left(\frac{1.9}{2}\right) = 1.71.$$

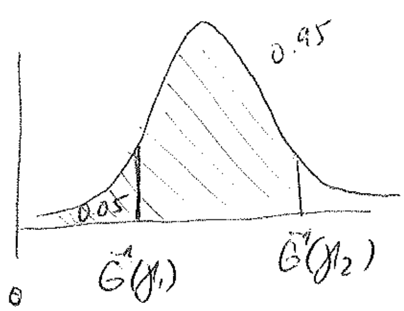
the observed conf interval for μ is $(3.68, 7.86)$.

c) $X_i \sim \exp(\lambda)$. Find a 90% conf interval for the mean monthly income.

$f(x_i) = \lambda e^{-\lambda x_i}$ and $E(X_i) = \frac{1}{\lambda}$ is the mean of the monthly income. So we want a 90% conf. interval for $\frac{1}{\lambda}$.

since $X_i \sim \exp(\lambda)$ we have that $X_i \sim \text{Gama}(1, \lambda)$ and independent. so, $\sum X_i \sim \text{Gama}(n, \lambda)$. Then,

$2\lambda \sum_{i=1}^{n} X_i \sim \chi^2_{(2 \cdot n)}$, $n = 25$.



$$P\left(G^{-1}(0.05) < 2\lambda \sum X_i < G^{-1}(0.95)\right) = \gamma = 0.9.$$

$$P\left(\frac{G^{-1}(0.05)}{2\sum X_i} < \lambda < \frac{G^{-1}(0.95)}{2\sum X_i}\right) = 0.9$$

$$a < x < b$$
$$\frac{1}{a} > \frac{1}{x} > \frac{1}{b}.$$

$$P\left(\frac{2\sum X_i}{G^{-1}(0.95)} < \frac{1}{\lambda} < \frac{2\sum X_i}{G^{-1}(0.05)} \right) = 0.9.$$

so under the exponentially distributed random variables assumption, a 90% conf. interval for the mean monthly income is given by

$$(A, B) = \left(\frac{2\sum X_i}{G^{-1}(0.95)} , \frac{2\sum X_i}{G^{-1}(0.05)} \right)$$, where

$G^{-1}(0.95)$ and $G^{-1}(0.05)$ are the 0.95 and 0.05 quantiles of a χ^2 distribution with $2n$ degrees of freedom.

the observed value of the interval is

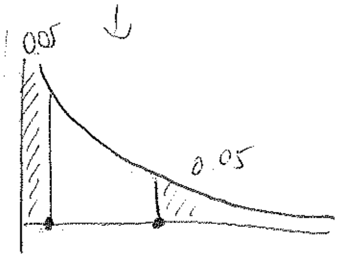
$$(a, b) = \left(\frac{2 \cdot 144.3}{67.50} , \frac{2 \cdot 144.3}{34.76} \right) = (4.2, 8.3)$$

d) $X_i \sim \exp(\lambda)$. $\lambda \sim Gama(\alpha, \beta)$, $\alpha = 1$, $\beta = 0.25$.

we need to find values c and d such that

$$P(c < \tfrac{1}{\lambda} < d) = 0.9.$$

Since $\lambda \sim Gama(\alpha, \beta)$, then $2\lambda \cdot \beta \sim \chi^2(2\alpha)$



$$P\left(G^{-1}(0.05) < 2\lambda\beta < G^{-1}(0.95) \right) = \mu.$$

$$P\left(\frac{G^{-1}(0.05)}{2\beta} < \lambda < \frac{G^{-1}(0.95)}{2\beta} \right) = \mu.$$

$$P\left(\frac{2\beta}{G^{-1}(0.95)} < \frac{1}{\lambda} < \frac{2\beta}{G^{-1}(0.05)} \right) = \mu = 0.9$$

$$c = \frac{2\beta}{G^{-1}(0.95)} = \frac{2 \cdot 0.25}{5.99} = 0.08$$

$$d = \frac{2\beta}{G^{-1}(0.05)} = \frac{2 \cdot 0.25}{0.11} = 4.87.$$

So with probability 0.9 the mean monthly income is between 0.08 and 4.87.